Tanta University		Department: **Computers and Control Engineering** Total Marks: ٧٠ Marks		Faculty of Engineering

Course Title: **Fundamentals of Stochastic Processes** أسس العمليات العشوائية   Course Code: CCE٣١١٧   ٣rd year
Date: ١٥.١.٢٠١٣ (First term)   Allowed time: ٣ hrs   No. of Pages: (٢)

Answer the following four questions. You are allowed to use the accompanying two tables of standard normal curve ordinates and areas in your answers.

Question No. ١   (١٦ marks)

(a) Let S={a, b, c, d, e, f} with P(a)=١/١٦, P(b)=١/١٦, P(c)=١/٨, P(d)=٣/١٦, P(e)=١/٤ and P(f)=٥/١٦. Let A={a, c, e}, B={c, d, e, f} and C={b, c, f}. Find:

    i) $P(A/B)$.
    ii) $P(B/C)$.
    iii) $P(C/A^C)$.
    iv) $P(A^C/C)$.

(b) Let A, B, and C be events. **Find an expression, and exhibit the Venn diagram,** for the event that:

    i) A and B, but not C occurs.
    ii) Only A occurs.

(c) In a certain college, ٢٥% of the boys and ١٠% of the girls are studying mathematics. The girls constitute ٦٠% of the students. If a student is selected at random and is studying mathematics, **determine the probability that the student is a girl?**

Question No. ٢   (١٨ marks)

(a) Find the expectation, variance, and standard deviation of the random variable x with density function P(x) given as:

x	١	٣	٤	٥
P(x)	٠.٤	٠.١	٠.٢	٠.٣

(b) Prove that for any random variable x:

    i) $E(ax + b) = a\,E(x) + b$
    ii) $V(ax + b) = a^2\,V(x)$
    iii) $E(c) = c$
    iv) $V(c) = 0$

where a, b, and c are constants.

(c) If the density function f(x) is given by:

$$f(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ x-1 & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

find the distribution function F(x).

Question No. ٣

(١٨ marks)

(a) A coin, weighted with P(H)= ٣/٤ and P(T)= ١/٤, is tossed three times. Let x be a random variable denoting the longest string of heads that occurs. **Find the distribution, expectation, variance, and standard deviation of x.**

(b) Consider the following binomial probability distribution:

$$P(x) = \binom{٥}{x}(٠.٧)^x (٠.٣)^{٥-x} \qquad (x = ٠, ١, ..., ٥)$$

where x is a random variable.
  i) How many trials (n) are in the experiment?
  ii) What is the value of p, the probability of success?
  iii) Graph P(x).
  iv) Find the mean and standard deviation of x.

(c) Suppose ٢% of items made by a factory are defective. **Find the probability that there are ٣ defective items in a sample of ١٠٠ items.**

Question No. ٤

(١٨ marks)

(a) Let x be a random variable with a standard normal distribution Φ. Find:
  i) $P(x \geq ١.١٣)$
  ii) $P(٠ \leq x \leq ١.٢٤)$
  iii) $P(٠.٦٥ \leq x \leq ١.٢٦)$
  iv) $P(-٠.٧٣ \leq x \leq ٠)$

(b) Let x be a random variable with the standard normal distribution Φ. **Determine the value of t, standard units, if:**
  i) $P(٠ \leq x \leq t) = ٠.٤٢٣٦$
  ii) $P(x \leq t) = ٠.٧٩٦٧$
  iii) $P(t \leq x \leq ٢) = ٠.١٠٠٠$

(c) A class has ١٢ boys and ٤ girls. If three students are selected at random one after the other from the class, **what is the probability that they are all boys?**

Best wishes

The Fundamentals of Stochastic processes

Sheet no.5

1) Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer .suppose five cancer patients are treated with this type of chemotherapy and let x equal the no. of successful cures out of the five .

x	0	1	2	3	4	5
P(x)	0.002	0.029	0.132	0.309	0.360	0.168

The probability distribution of x is given in the following table.

Find:

    a) $\mu = E(x)$

    b) $\sigma = \sqrt{E(x-\mu)^2}$

2) Find the expectation, variance .and the standard deviation of each of the following:

i)

x	2	3	11
P(x)	1/3	1/2	1/6

ii)

x	-5	-4	1	2
P(x)	1/4	1/8	1/2	1/8

iii)

x	1	3	4	5
P(x)	0.4	0.1	0.2	0.3

iv)
$$p(x) = \begin{cases} \dfrac{2}{25}x & 0 \le x \le 5 \\ 0 & elsewhere \end{cases}$$

3) Prove for any random variable x

    I) $E(ax+b) = aE(x) + b$

    ii) $V(ax+b) = a^2 v(x)$

    iii) $E(c) = c$

    iv) $V(c) = 0$

4) The heart association claims that only 10% of adults over 30 can pass the physical fitness test. Suppose that four adults are randomly selected and each is given the fitness test.

    a) Find the probability that three of the four adults pass the test *none*

    b) Find the probability that three of the four adults pass the test

    c) Let x represent the number of the four adults who pass the test

    d) Drive a formula for p(x), the probability distribution of the binomial random variable x.

5) Refer to problem 4. Use the formula for a binomial random variable to find the probability distribution of x, where x is the number of adults who pass the fitness test, graph the distribution

x	0	1	2	3	4
P(x)	0.6561	0.2916	0.0486	0.0036	0.0001

6) Refer to problem 5 .Calculate the mean and the standard deviation.

7) Give a formula for p(x) for a binomial random variable with n=7 and p= 0.2

8) Consider the following binomial probability distribution

$$P(x) = \binom{5}{x}(0.7)^x(0.3)^{5-x}, \quad X = 0, 1, 2, 3, 4, 5$$

a) How many trials n are in the experiment?

b) What is the value of p .the probability of success?

c) Graph p(x)

d) Find the mean and the standard deviation of x.

9) Suppose X is a binomial random variable with n = 3 and p = 0.3

a) Calculate the value of p(x), x=0, 1, 2, 3, using the formula for a binomial probability distribution.

b) Find the mean and the standard deviation of x

10) If x is a binomial random variable. Calculate mean, variance and standard deviation for each of the following

a) n =80   ,p=0.2

b) n =70   ,p=0.9

c) n =1000 , p=0.04